Reassessing D8 Optimization Graphically

D8: I can use calculus and derivative tests to find extreme values and classify them as maxima or minima.

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
   (a) not understanding a concept;
   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
   (d) not knowing how to start or formulate an approach to the problem;
   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

Where topic was first introduced: Module 10

Basic Principles

- Critical points: This homework problem explored the subtleties of the definition. Remember that until you have memorized the definition it is a good idea to write it down from the source.

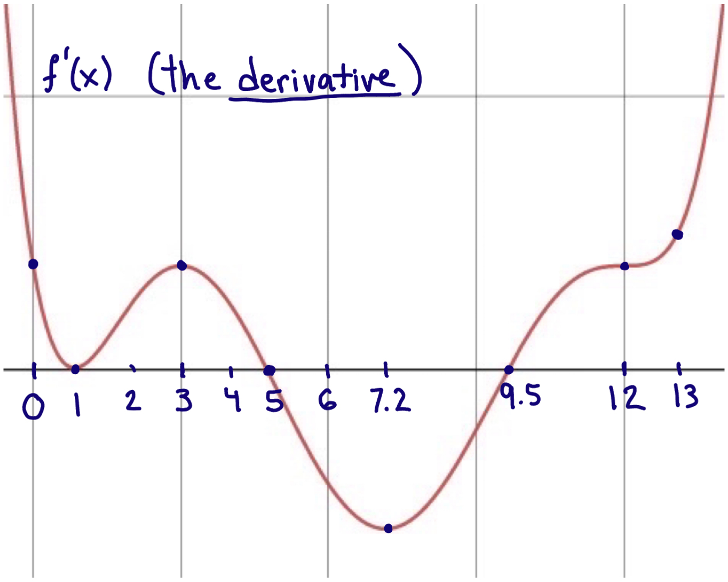
  Critical point - We say a function, f, has a *critical point* at $x = p$ if p is **in the domain** of f and $f'(p) = 0$ OR $f'(p)$ is undefined. The point $(p, f(p)$ on the graph of f is also called a *critical point.*

  ("in the domain" means that the parent function exists at that point, or that $f(p)$ exists.)

  There are three basic parts to check: domain, $f' = 0$, and f' DNE. You must check for BOTH where f' is 0 or undefined, OR STATE why there are no x values where f' is 0 or undefined. Otherwise once you start testing your results could be flawed.

- Extreme values can only happen at end points and places where the function might change direction (switch from increase to decrease or vice versa.)
- Functions **might** change direction when the derivative is neither positive nor negative, so when $f'(x) = 0$ or $f'(x)$ DNE. Between any two consecutive critical points the function CANNOT change direction because if it did there would have been another critical point there. Thus if the function is increasing at ONE x value between critical points, the function is increasing for ALL values between critical points.
- First Derivative Test: use intervals of increase or decrease to determine if a function switches from increasing to decreasing (local max) or decreasing to increasing (local min) or neither at critical points. Can also be used to classify endpoints: for example, if a function decreases to the right of the left endpoint, then the left end point must have been higher than nearby values and thus must be a local max.
- Second derivative test: IF your critical point corresponds to a horizontal tangent line (zero slope) and IF f'' is not zero, then you can use concavity to determine extrema: Let cp be a critical point. If $f'(cp) = 0$ and $f''(cp) > 0$, then there is a local minimum when $x = cp$. If $f'(cp) = 0$ and $f''(cp) < 0$, then there is a local maximum when $x = cp$. Otherwise this test is inconclusive or doesn't help us.

Example:



- Because this graph shows the derivative, and is continuous, we can see that $f'(x)$ is never undefined, so we only need to look for critical points where $f'(x) = 0$, so our critical points are 1, 5 and 9.5.
- Because $f'(x)$ is positive on both sides of $x = 1$ we do not have a max or min. Because $f'(x)$ changes from positive to negative when $x = 5$, f must have a local maximum.
- Because $f'(x)$ is increasing when $x = 9.5$ we know that $f''(9.5) > 0$ therefore $f(9.5)$ gives us a local minimum by the second derivative test.

Favorite Mistakes:

- Misinterpreting the graph- is the image of the parent function (f) or the first derivative (f') or the second derivative (f'')?
- Not understanding what the second derivative test does. It's for classifying a critical point as a max or min using concavity. If you find where $f'' = 0$ and then test on either side of that, you might be confirming an inflection point, which does NOT help with finding if we have a max or min.
- Testing either side of an x value with the second derivative. The second derivative test needs to know the concavity AT the critical point.
- Favorite mistake: just stating that $f'' > 0$ instead of explaining WHY.

Prepare for Revision:

Sketch a graph of a function, making sure it crosses the $x-$axis at least 3 times. Call this function the first derivative and use it to find and classify all of your critical points of the parent function, f, as max/min or neither, and be sure to use both the first and second derivative tests.